the elastic moduli at zero field and pressure are shown in Fig. 2. Data published by Rosen and Klimker [9] show similar, but significantly different, effects. The differences between Fig. 2 and Rosen's data are primarily in the variation of  $c_{12}$ ; consequently, the variations of the compressibilities  $\beta_{\parallel}$  and  $\beta_{\perp}$  with turn angle and structure, that are calculated from the Fig. 2 data, are considerably different than given by Rosen. In the present context, however, there appears no indication that magnetic ordering has any influence on the second order elastic moduli at 298°K.

The magnetic structures in Er are even more complex. The spontaneous ferromagnetic phase, with  $T_c = 20^{\circ}$ K, contains a spiral component. The ferromagnetic component transforms to an antiferromagnetic arrangement, between 20 and 53°K, and a modulated moment arrangement between 53 and 80°K. The effects of the ordering on the elastic moduli are shown in Fig. 3. These effects are clearly not evident at  $298^{\circ}$ K.

The values of the second order elastic stiffness moduli at 298°K are listed in Table 2. We include here the data for Ho as given by Palmer and Lee[8]. Included in Table 2 are the values of the density[18] that are used for computing the moduli. The calculated adiabatic and isothermal bulk moduli,  $K_s$  and  $K_T$ , and also given and the last column gives the parameters  $(\beta_{\parallel} - \beta_{\perp})/\beta_{V}$  calculated from isothermal values of the compressibilities. The latter parameter is that used for computing equation (3). The variations of the data with increasing atomic number (Gd  $\rightarrow$  Er) are noteworthy. The c/a ratio decreases whereas the density increases because of the so-called lanthanide contraction associated with the addition of electrons to the 4f shell. There is



Fig. 2. Temperature dependence of the principal elastic moduli of Dy at zero magnetic field.  $T_N = 179^{\circ}$ K and  $T_c = 87^{\circ}$ K. Absence of data for  $c_{11}$  and  $c_{66}$  at  $T < 85^{\circ}$ K is result of spontaneous macroscopic distortion by anisotropic magnetostriction.



Fig. 3. Temperature dependence of principal elastic moduli in Er at zero magnetic field, where magnetic phase changes occur at  $\sim 20, \sim 53$  and 80°K. Dashed lines indicate thermal hysteresis in  $T_c \sim 20$ °K. Macroscopic distortion prevented measurements of  $c_{11}$  and  $c_{66}$  in spiral phase.

Table 2. Values of second order elastic moduli at 298°K of heavy rare earth metals (Kbars)

	c/a	Density	<i>c</i> <sub>11</sub>	C <sub>33</sub>	C44	C <sub>66</sub>	C <sub>H</sub>	<i>C</i> <sub>12</sub>	C <sub>13</sub>	K <sub>s</sub>	K <sub>T</sub>	$\left(\frac{\beta_{  }-\beta_{\perp}}{\beta_{V}}\right)_{T}$
Gd	1.590	7.888	667	719	207	208	250	250	213	378	373	+0.007
Dy(F)	1.573	8.560	747	787	243	243	278	262	223	411	410	-0.005
DV(R)		8.545	743	790	255	246	290	251	208	402	401	-0.009
Dv(P)		8.560	731	781	240	239	276	253	223	410	409	-0.015
Ho	1.570	8.800	761	776	257	256	290	248	206	401		+0.019
Er	1.569	9.064	863	855	281	279	328	305	227	455	450	+0.043

a general increase with atomic number in all of the  $c_{ij}$  and K values, with some exceptions for the Ho data.

## (b) Variations of $c_{ij}$ with P at 298°K

Plots of the data for Dy and Er showed no significant or systematic departure from linear relations between the  $c_{ij}$  and pressure. The Gd data showed small but significant curvature

for the  $c_{33}$  vs *P* plot at P > 2 Kbar and for  $c_{44}$  at P = 0 and 2 Kbar. All of the data were, however, analyzed by least square statistics to obtain the most probable values for the  $dc_{ij}/dP$ , assuming no curvature. The results with indicated probable errors are given in Table 3. The similarities and differences in the effects of pressure on the  $c_{ij}$  of the three metals are evident in the pressure derivatives